Three-body models of light nuclei with nonlocal potentials

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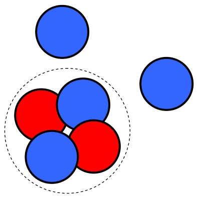
Three-body models

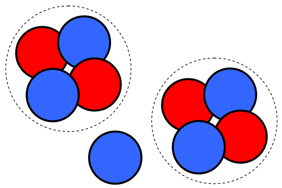
Two approaches :

3-cluster microscopic models

^{6}He: αnn



^{9}Be: $\alpha\alpha$n



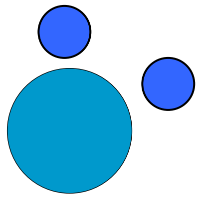
Involving all nucleon coordinates

+ Pauli Principle between nucleons

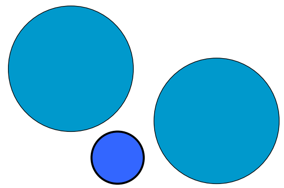
Heavy computational work !

3-body non-microscopic models

^{6}He: αnn



^{9}Be: $\alpha\alpha$n



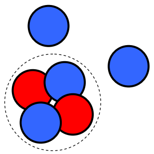
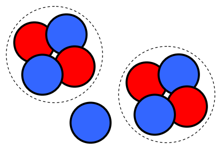
Simplification:

Structureless clusters

Need for αn and $\alpha\alpha$ potentials !

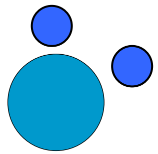
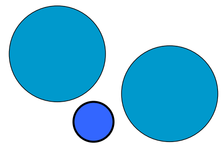
Two approaches :

3-cluster microscopic model

^{6}He  ^{9}Be 

all the nucleons + Pauli principle

3-body non-microscopic model

^{6}He  ^{9}Be 

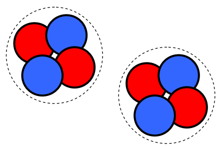
αnn $\alpha\alpha n$

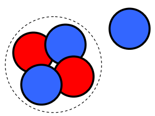
need for $\alpha\alpha$ and αn potentials

A simpler problem :

→ **2 clusters**

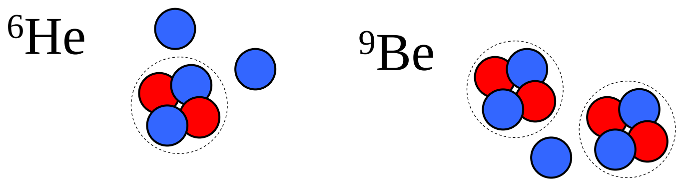
2-cluster microscopic model: RGM

$\alpha\alpha$  ⇒ non-local $\alpha\alpha$ potential

αn  ⇒ non-local αn potential

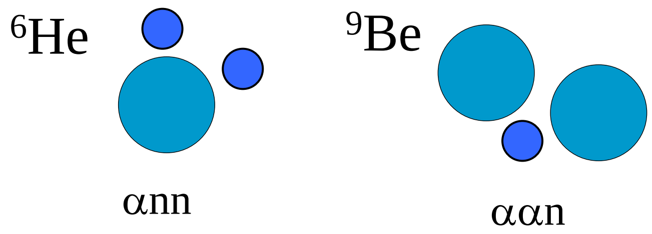
Non-local potentials because of the Pauli principle

3-cluster microscopic model



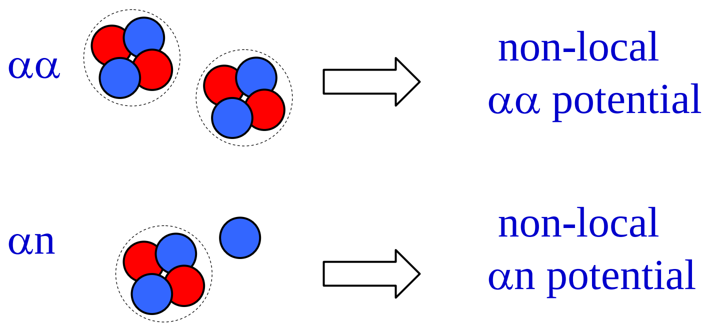
all the nucleons + Pauli principle

semi-microscopic model



3-body model using non-local RGM potentials

2-cluster microscopic model: RGM



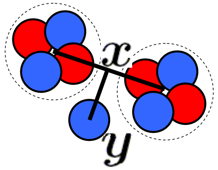
Is the semi-microscopic model a reasonable approximation of the microscopic model ?

Comparison between models

Microscopic model : $H\Psi = E\Psi$

all the nucleons: $$H = \sum_{i=1}^{A} t_i + \sum_{i>j=1}^{A} v_{ij}$$

3 clusters :
Pauli principle $$\Psi = \mathcal{A}\,\phi_1\phi_2\phi_3\, g(\boldsymbol{x},\boldsymbol{y})$$



Model assumptions :

- ϕ_i in HO shell model,
- v_{ij} : effective nucleon-nucleon potential
  (Minnesota or Volkov 2)

Generator-Coordinate Method (GCM) in the hyperspherical formalism

S. Korennov, P. Descouvemont, Nucl. Phys. A 740 (2004) 249

Semi-microscopic model :

Three-body model: $$\left(\sum_{i=1}^{3} T_i + \sum_{i>j=1}^{3} V_{ij}\right)\psi = E\,\psi$$ with $V_{ij} = V^{RGM}$

RGM under the same assumptions

(2-cluster microscopic model)

Solving 3-body Schrödinger equation with non-local potentials

using hyperspherical harmonics: *M. Theeten, D. Baye, P. Descouvemont,*
Nucl. Phys A 753 (2005) 233; Phys. Rev. C 76 (2007) 054003

Comparison between microscopic and semi-microscopic models

^{9}Be: $\alpha\alpha$n ^{6}He: αnn

nucleon-nucleon potential :

Minnesota (MN) or Volkov 2 (V2) + spin-orbit potential

→ (α+n and α+α scattering phase shifts)

Comparison :
- Binding energies
- Nuclear radius : $\langle r^2 \rangle^{1/2}$
- Neutron density : ρ_n

^{9}Be: ααn

Ground state : 3/2^{-}

NN potential: MN

	Microscopic model E (MeV)	Microscopic model $\langle r^2\rangle^{1/2}$ (fm)	Semi-microscopic model E (MeV)	Semi-microscopic model $\langle r^2\rangle^{1/2}$ (fm)	
reproducing α+n and α+α scattering phase shifts :	-2.61	2.36	-2.16	2.41	ΔE = 0.45 MeV
fitted to E_{exp} = -1.57 MeV : (exchange parameter of **MN**)	-1.57	2.43	-1.18	2.49	ΔE = 0.39 MeV
	-1.97	2.40	-1.57	2.46	ΔE = 0.40 MeV

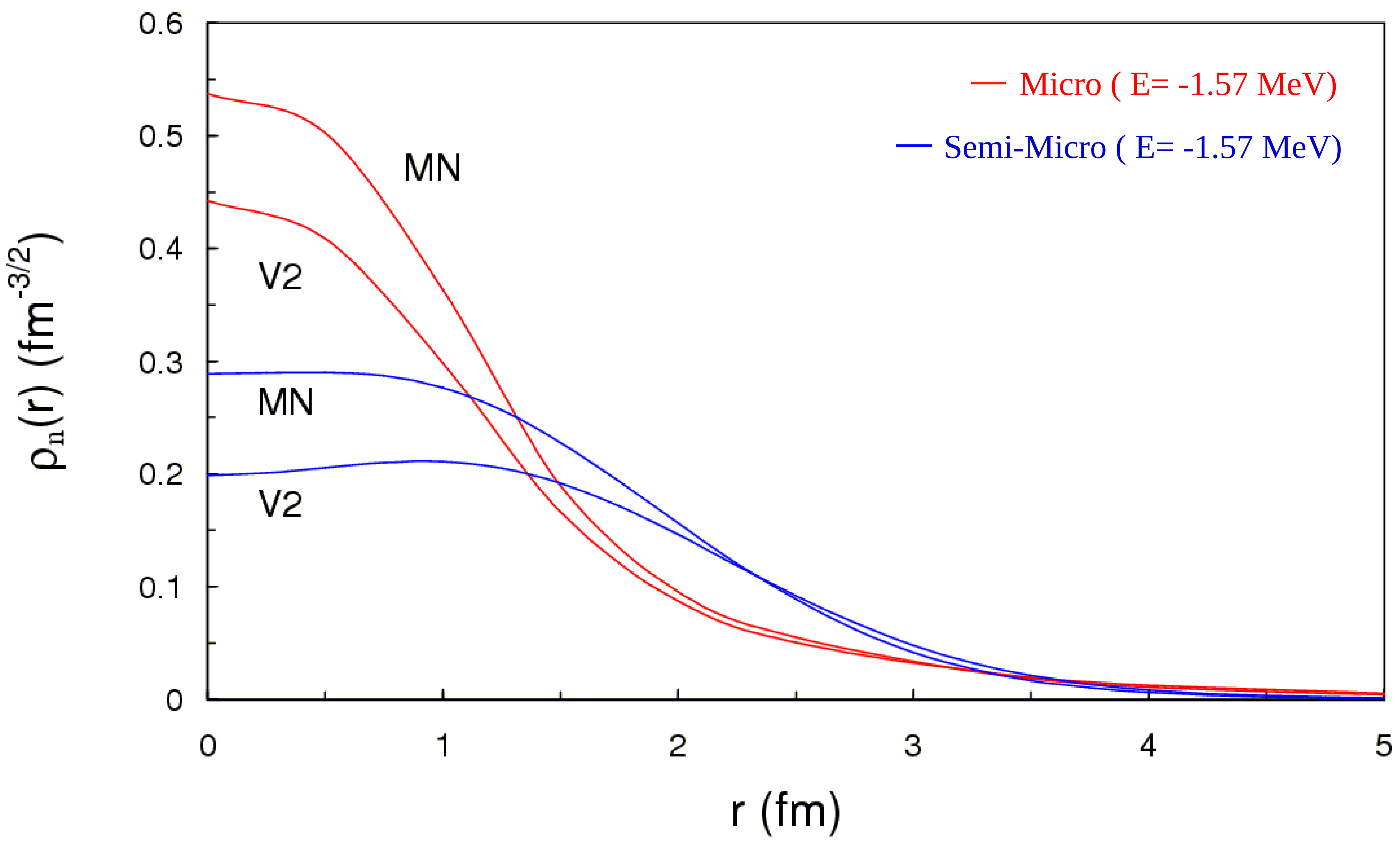
MN : ΔE ~ 0.4 MeV

NN potential: V2

	Microscopic model E (MeV)	Microscopic model $\langle r^2\rangle^{1/2}$ (fm)	Semi-microscopic model E (MeV)	Semi-microscopic model $\langle r^2\rangle^{1/2}$ (fm)	
reproducing α+n and α+α scattering phase shifts :	-1.36	2.60	-1.12	2.41	ΔE = 0.24 MeV
fitted to E_{exp} = -1.57 MeV : (exchange parameter of **V2**)	-1.57	2.58	-1.32	2.66	ΔE = 0.25 MeV
	-1.81	2.56	-1.57	2.63	ΔE = 0.24 MeV

V2 : ΔE ~ 0.24 MeV

Neutron densities: $\alpha\alpha n$ system (^{9}Be)



^{6}He: αnn

Ground state : 0^+

NN potential: MN

reproducing α+n and $\alpha+\alpha$ scattering phase shifts :

fitted to E_{exp} = -0.974 MeV : (exchange parameter of **MN**)

Microscopic model		Semi-microscopic model		
E (MeV)	$\langle r^2\rangle^{1/2}$ (fm)	E (MeV)	$\langle r^2\rangle^{1/2}$ (fm)	
-0.07	2.57	-0.08	2.93	ΔE = -0.01 MeV
-0.98	2.38	-0.95	2.46	ΔE = 0.03 MeV
-1.02	2.37	-0.97	2.46	ΔE = 0.05 MeV

MN : $\Delta E \sim 0.05$ MeV

NN potential: V2

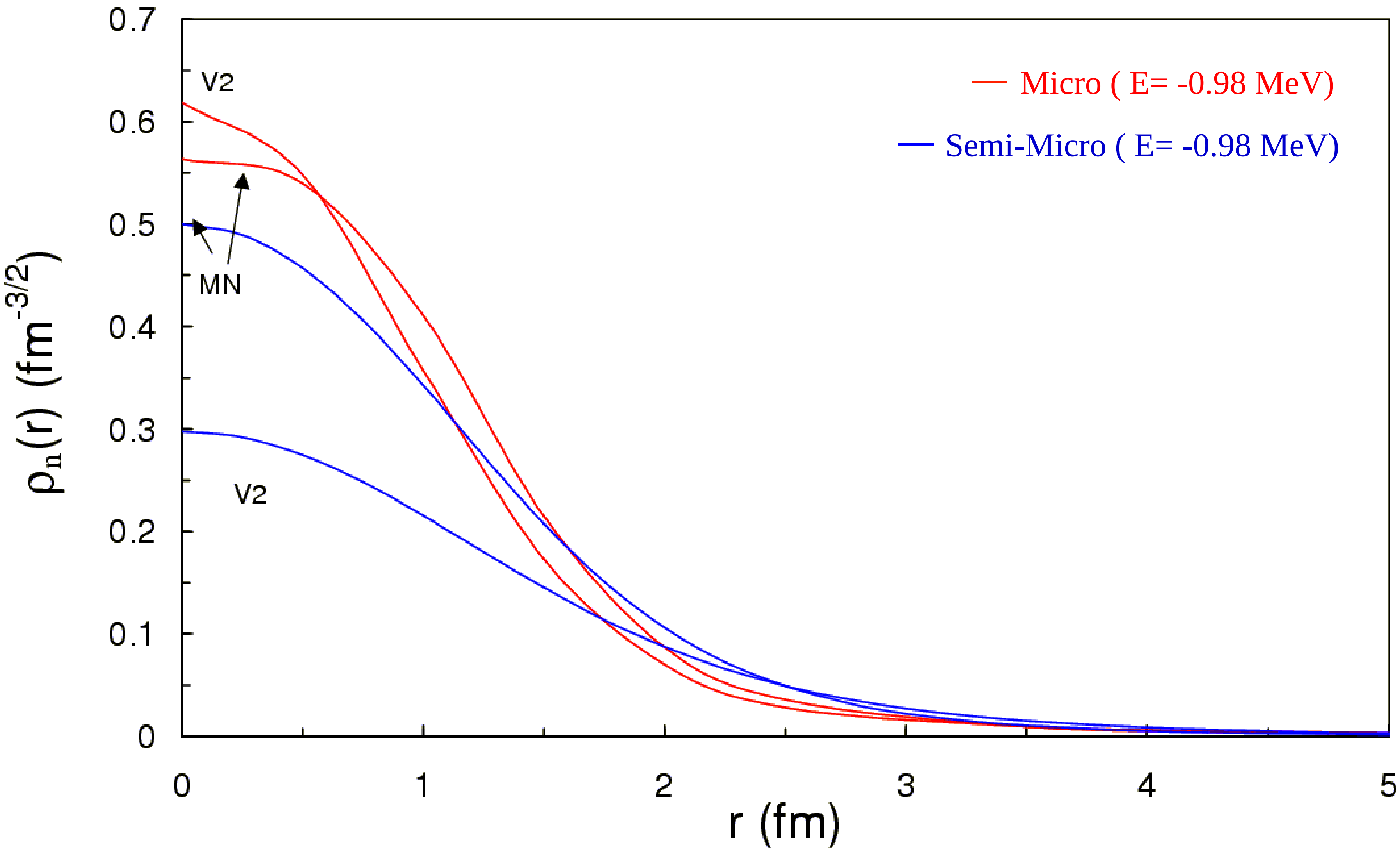
reproducing α+n and $\alpha+\alpha$ scattering phase shifts :

fitted to E_{exp} = -0.974 MeV : (exchange parameter of **V2**)

Microscopic model		Semi-microscopic model		
E (MeV)	$\langle r^2\rangle^{1/2}$ (fm)	E (MeV)	$\langle r^2\rangle^{1/2}$ (fm)	
-2.43	2.46	-1.96	2.56	ΔE = 0.47 MeV
-0.98	2.90	-0.65	4.11	ΔE = 0.33 MeV
-1.43	2.74	-0.98	3.23	ΔE = 0.45 MeV

V2 : $\Delta E \sim 0.4$ MeV

Neutron densities: αnn system (^{6}He)



Conclusion

Semi-microscopic model :

3-body model using non-local RGM potentials.

(from 2-cluster RGM (microscopic model))

Pauli principle

^{9}Be ($\alpha\alpha$n) , ^{6}He (αnn)

The semi-microscopic model is an approximation of the 3-cluster microscopic model.

Comparison

- Binding energies: underestimated by semi-microscopic approaches.

  ((this means) *3-body exchange effects* (missing in this approximation) should be *attractive*.
  Pauli principle)

- Nuclear densities:

  Densities obtained with the semi-microscopic model are lower at short distances.

approximation: better for the light nuclei : very good agreement for ^{6}He (with MN interaction)

aslo, (α+n and α+α phase shifts $\not\Rightarrow$ ^{9}Be ($\alpha\alpha$n) , ^{6}He (αnn)) … 3-body effects.